# A Neuroeconomic Theory of Memory Retrieval *

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## Abstract

We propose a theory of "optimal memory management" that unveils *causal* relationships between memory systems and the characteristics of the information retrieved. Our model shows that if the declarative memory is more accurate but also more costly than the procedural memory, then it is optimal to retrieve exceptional experiences with the former and average experiences with the latter. The theory provides other testable predictions: (i) decisions are closer to original experiences when the declarative memory is invoked, and (ii) the declarative memory is more likely to be invoked when the importance of recalling information accurately increases.

Keywords: memory systems, memory management, declarative, procedural, neuroeconomic theory.

JEL classifications: D01, D83, D87.

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# 1 Introduction

Bounded memory is arguably one of the most important limitations in humans, an aspect that has received considerable attention from researchers. Formal mathematical models of limited memory can be found in fields as diverse as statistics (Cover and Hellman, 1970), artificial intelligence (Narendra and Thathachar, 1989), psychology (Anderson and Milson, 1989) and computation theory (Feder, 1991), just to name a few. There is also a literature in economics (see e.g., Piccione and Rubinstein (1997), Mullainathan (1998), Benabou and Tirole (2002), Bernheim and Thomadsen (2005), Frey (2005), Kocer (2012) and Monte (2014)), although it is fair to say that it has received less attention than other aspects of bounded rationality.

Bounded memory is multifaceted. In order to concentrate on some aspects, all the formal models we know have overlooked one important finding in neuroscience: memories can be encoded by different systems and each system has some special properties. The goal of this paper is to build the (to our knowledge) first model of bounded memory in economics or any other science where experiences are *optimally encoded by different systems* depending on their characteristics. More precisely, our model unveils causal relationships between the memory system employed and the type of the information retrieved. To better understand the building blocks of the theory, we first present a brief overview of the existing neurophysiological evidence on memory (these findings are well-known in neuroscience but possibly less familiar for economists).

Memory refers both to the conscious recollection of facts and historic events and also to the unconscious and automatic retrieval of information necessary to perform some habitual actions. However, the processes involved in storing, learning and retrieving these different types of information differ largely. The literature in neuroscience reports findings indicating the existence of different memory systems in the brain (see e.g. Poldrack and Foerbe (2008) for a review). An accurate classification of memory systems has been obtained by correlating the types of information memorized with the underlying biological mechanisms involved in the memory processes (see e.g. Squire (2004) for a review). Memory can be broadly classified into two main classes.

*Declarative memory* refers to the capacity to recollect information in a conscious way. It is based on the ability to detect and encode what is unique about an event (Ullman, 2004). Learning occurs fast (with few exposures) and the learned material is consciously known and easily verbalized. Learning is effortful and engages working memory resources (Craik et al., 1996). The knowledge acquired with the declarative memory system is flexible and can be used in a variety of contexts, but it also tends to erode. Declarative memory engages the hippocampus and surrounding structures. These structures are involved in the formation of memories but also in the ability to retain and recall them (Gabrieli and Kao, 2007). The lateral Prefrontal cortex (lPFC) is engaged in the memory process of contextual details of an experience. The left dlPFC is activated when memories are formed while the right dlPFC is activated when memories are retrieved (Kapur et al., 1997) and these structures are also more active during the encoding of unexpected facts (Fletcher et al. (2001)). The amygdala is involved in the encoding and retrieval of emotionally charged memories (Adolphs et al., 1997).

*Non-declarative memory* refers both to learned skills and habits and perceptual learning or conditioning. Non-declarative memory detects what is common to several situations. Learning is gradual and slow, the decision-maker learns through trial-and-error, and requires feedback. The learned material is also unconscious and difficult to verbalize. Learning requires effortless attention. Learned knowledge is rigid, used in specific contexts, and durable. It engages a variety of structures depending on the finer subclassification of memories. Closest to the specific interest in this article, the part of the non-declarative memory that refers to skills and habits is placed under the umbrella of *procedural memory*. It engages structures like the striatum (Kreitzer, 2009). Also, conditioning is linked to the amygdala and the cerebellum (see Squire (2004) for a detailed classification).

This classification suggests a tight connection between memory system and type of information. We can think of the different systems as tools to solve different problems. For instance, the declarative system helps find a solution to problems like "in which spot did I park today?" while the procedural system solves best problems like "when I come to school where do I usually park?". A summary of the major differences between the declarative and procedural memory systems is presented in Table 1.

| **Memory System** | **Declarative** | **Procedural** |
|---|---|---|
| Characteristics | Fast and conscious<br>Flexible and temporary<br>Effortful<br>Precise | Slow and unconscious<br>Rigid and durable<br>Effortless<br>Vague |
| Uses | Facts and events<br>Unique features | Skills and habits<br>Common features |
| Brain areas | Medial temporal lobe<br>(hippocampus) | Basal ganglia<br>(striatum) |

**Table 1**. Taxonomy of declarative and procedural memory systems

The relationship between memory systems and types of memories is still imperfectly understood. Yet, existing studies provide interesting findings. Firstly, memory systems are *substitutable*. Bayley et al. (2005) show that subjects with impaired procedural memory improve over time their performance in the weather prediction task by repeatedly exercising their declarative memory, even though this is a paradigmatic example where procedural memory works best. Also, what is learned depends crucially on which system is engaged (Dagher et al., 2001). Overall, systems are tailored to certain types of memories and act as 'imperfect substitutes' (Poldrack and Packard, 2003). Secondly, systems are selected depending on task demands. In particular, there is evidence that neurobiological mechanisms are in place to make sure behavior is *optimized*, that is, it employs the memory system most suitable to the experience (Poldrack et al. (2001); Foerde et al. (2006)).

Substitutability and optimization are key properties in decision making. The evidence reviewed here suggests that the resort to a given memory system is an endogenous decision: (i) several systems can be employed to retrieve memories, (ii) different systems have different properties which make them suitable for the encoding and retrieval of different experiences, and (iii) the choice of one system over another will be the result of an optimization process. Starting from these premises, the purpose of this study is to build a theory of optimal memory management, that is, one that predicts the choice between competing memory systems as a function of the experience to memorize.

Combining the findings reported above, we build a simple model in which a decision-maker (hereafter DM) learns a piece of information relevant for future choices. DM has imperfect memory, so the exact information received may not be correctly recalled at the time of the future decision. The information is stored and retrieved using either the declarative memory system or the procedural memory system. We work under the hypothesis that these systems differ in their *accuracy* and *cost*. Accuracy corresponds to the degree to which DM can recover the precise experience, while cost refers to the attentional resources needed to encode and retrieve the information. As summarized in the second row of Table 1, the declarative memory system produces more accurate representations of the experience but is also more costly than the procedural memory system because it requires more attention to process contextual and emotional information (Craik et al., 1996).[1] We then show that some of the situations emphasized in the literature where different systems are employed (the third row of Table 1) are precisely the ones predicted by our theory under an optimal memory management strategy.

## 2 A simple model of memory retrieval

To formalize our thought experiment, we consider a four-stage decision-making problem.

In stage 1, DM acquires information about the state of the world. Let $x \in \mathbb{R}$ be the state learned by the individual, and denote by $X$ the real-valued random variable from

[1] Although these systems differ in many other respects as well and some other systems are also at play in the storing and retrieval of memories, our theory focuses exclusively on those two characteristics in order to better assess their impact on behavior.

which $x$ is drawn. We assume that $X$ follows a normal distribution. Formally:

$$X \sim \mathcal{N}\left(\mu, \tfrac{1}{p}\right),$$

where $\mu$ is the mean and $p$ the precision (inverse of variance) of the random variable $X$.

In stage 2, a memory about the state is formed. DM can invoke the declarative memory system ($i = D$) or the procedural memory system ($i = P$). This choice impacts future memory recollections.

In stage 3, the state is noisily recollected. If memory system $i$ ($\in \{D, P\}$) was invoked during the encoding phase, the individual retrieves the following signal $s_i$ correlated with the true state $x$:

$$s_i = x + u_i \qquad \text{where} \quad u_i \sim \mathcal{N}\left(0, \tfrac{1}{h_i}\right).$$

The noise $u_i$ follows a normal distribution with mean 0 and precision $h_i$. In expectation, the signal is correct $E(s_i) = x$. In order to capture the greater accuracy of information retrieval under the declarative system than under the procedural system, we assume that $h_D > h_P$ (notice that $h_i = +\infty$ implies perfect recollection of the state whereas $h_i = 0$ implies no recollection whatsoever). Also, invoking memory system $i$ has a cost $c_i$. Following the evidence previously described, we assume that the declarative system involves a higher cost than the procedural system, namely $c_D > c_P$.

In stage 4, DM takes an action $a$ and his payoff depends on the congruence between the action and the state. For simplicity, we assume that the payoff $l(a, x)$ is given by a standard quadratic utility loss:

$$l(a, x) = -\beta\,(a - x)^2$$

with $\beta > 0$. According to this formulation, if the state $x$ is recalled with exactitude, the individual's optimal action is:

$$\tilde{a}(x) = \arg\max_a \; l(a, x) \quad \Rightarrow \quad \tilde{a}(x) = x.$$

Deviations from $\tilde{a}(x)$ imply a loss which is increasing in $\beta$. The parameter $\beta$ thus represents the importance of the decision or the sensitivity of DM to losses.

A simple example to illustrate this sequence of events is that of a DM recalling how much he liked a product before purchasing it again. His experience in stage 1 reveals the optimal quantity he should purchase (the state, $x$). In stage 3, he forms a recollection of the experience (the signal, $s_i$) but it will be distorted due to imperfect memory (the noise, $u_i$). Based on such recollection, DM in stage 4 may decide to purchase too much or too little of the product (the action, $a$), that is, $a \gtrless x$. Both deviations imply a utility loss.

Notice that the declarative system is tailored to answer the question "how much do I like this particular product?". If invoked, it is likely that DM will recall a signal $s_D$ close to the experience $x$ and answer the question correctly. However, precision comes at a cost, and working memory is highly involved in the process. On the other hand, the procedural system is designed to answer the more general question "how much do I like this type of products?". If it is engaged, the memory recollection $s_P$ is likely to be farther away from the experience $x$, as DM will miss specific features of the product and focus instead on general characteristics he typically likes. The benefit, however, is that this memory retrieval requires little effort.

The decision process is summarized by the following timeline.

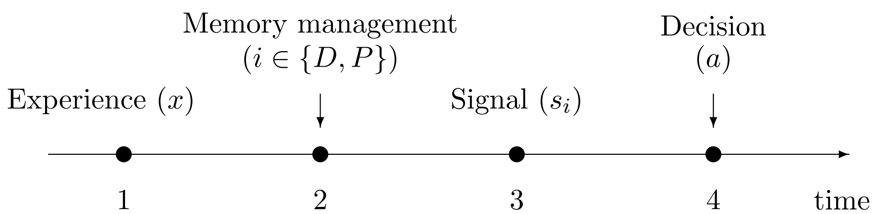


**Figure 1**. Timing

The problem is solved by backward induction. If memory system $i$ is chosen in stage 2 and signal $s_i$ is retrieved in stage 3, then DM in stage 4 should choose the action that maximizes his expected payoff:

$$\hat{a}(s_i) = \arg\max_{a} \; - \int_x \beta\,(a - x)^2 \, dF_i(x \,|\, s_i)$$

where $F_i(x \,|\, s_i)$ is the revised distribution of the state $x$ given the memory system $i$, the signal retrieved $s_i$ and the prior distribution of states $X$. Since the objective function is quadratic, the optimal action satisfies the first-order condition, namely:

$$\hat{a}(s_i) = E_i[X \,|\, s_i],$$

where $E_i$ is the expectation operator. In words, DM's optimal action is simply his expected belief about the state given the signal retrieved.

In stage 2, after observing the state $x$, DM knows that if system $i$ is invoked, he will obtain in stage 3 a signal $s_i$ drawn from $G_i(s_i \,|\, x)$. Given that signal, he will undertake in stage 4 the action $E_i[X \,|\, s_i]$. Therefore, DM's expected payoff in stage 2 is:

$$V_i(x) = - \int_{s_i} \beta\,(E_i[X \,|\, s_i] - x)^2 \, dG_i(s_i \,|\, x) - c_i$$

The normality assumptions of state and signal imply that:

$$s_i \,|\, x \;\sim\; \mathcal{N}\left(x, \tfrac{1}{h_i}\right) \qquad \text{and} \qquad X \,|\, s_i \;\sim\; \mathcal{N}\left(\tfrac{p}{p+h_i}\,\mu + \tfrac{h_i}{p+h_i}\,s_i, \tfrac{1}{p+h_i}\right)$$

Substituting in the previous equation, we get:

$$V_i(x) = -\frac{\beta h_i}{(p+h_i)^2} - \frac{\beta p^2}{(p+h_i)^2}(x-\mu)^2 - c_i, \quad i \in \{D, P\} \tag{1}$$

Finally, in stage 2 it is optimal to employ the memory system $i$ that achieves the highest expected payoff, namely:

$$i^* = \arg \max_{i \in \{D,P\}} \; V_i(x)$$

# 3 Results

We can now review the properties of DM's decision in stage 4 given the memory system invoked, as well as DM's efficient memory management strategy in stage 2.

**Result 1** *DM's optimal action in stage 4 moves in the direction of the signal. It is close to the prior when the memory system employed is very imprecise (procedural) and close to the signal when the memory system employed is very precise (declarative).*

Under either system, the posterior belief about the state, and therefore about the optimal action to be taken, is a convex combination of prior $\mu$ and signal $s_i$. Formally, $\hat{a}(s_i) = \frac{p}{p+h_i}\mu + \frac{h_i}{p+h_i}s_i$. A signal below the prior indicates the state is likely to be below $\mu$, hence an optimal action $\hat{a}(s_i) \in (s_i, \mu)$. A signal above the prior indicates the state is likely to be above $\mu$, hence an optimal action $\hat{a}(s_i) \in (\mu, s_i)$. As the precision of the memory system increases, the signal $s_i$ becomes more informative and reliable. DM is then willing to put a higher weight on the signal and a lower weight on the prior.

Result 1 can be used to predict the direction of the choices under either memory system. Because of imperfect retrieval of the experience, those choices will imply some expected losses and an efficient memory management opts for the system yielding higher expected utility. We now present the main result of the paper, namely the characterization of the optimal choice of system.

**Result 2** *It is optimal to retrieve information with the declarative memory system when the state is extreme and with the procedural memory system when the state is intermediate.*

Formally, there exists a cutoff $x^* \geq 0$ such that $V_P(x) > V_D(x)$ if $x \in (\mu - x^*, \mu + x^*)$ and $V_D(x) \geqslant V_P(x)$ if $x \notin (\mu - x^*, \mu + x^*)$.[2] The result comes from equation (1)

[2] For some parameter configurations $(h_D, h_P, c_D, c_P)$, we have $V_D(x) > V_P(x)$ for all $x$.

and has an intuitive explanation. Since the information retrieved with the declarative system is precise, it puts a higher weight on the signal (thus a lower weight on the prior) compared to what the procedural system would achieve. When the state is close to the prior $\mu$, the utility loss of remembering it imperfectly is, on average, smaller the higher the weight put on the prior in forming the posterior. Hence, the procedural system is preferred. Conversely, when the state is far away from $\mu$, it is on average better to put a low weight on the prior and a high weight on the signal. Here the declarative system is preferred. Stated differently, because the declarative system encodes and retrieves more accurate information but at a higher cost, it is efficient to resort to that channel only when the true state departs substantially from the prior belief, and is therefore worth remembering accurately. Overall the declarative system is used only when states are extreme. This optimal memory management policy is illustrated in Figure 2. The policy can be implemented by a simple algorithm in which the distance between the true state and the prior belief is first established, and this determines which system should encode the memory about the state.

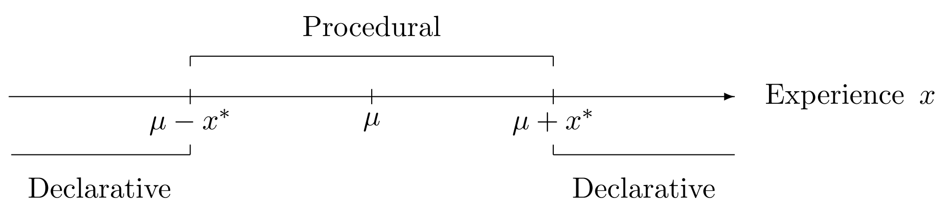


**Figure 2**. Optimal memory management

# 4 Implications of the theory

Our previous results establish that efficient memory management requires to use the declarative system when memories are worth remembering with precision, while the others should be managed by the procedural system. This has a series of implications.

*1. Remembering events differently as a function of how they strike us.*

According to our theory, DM will remember with accuracy an extremely good or bad experience with a product because it will engage the declarative memory system. However, an average experience will not stand out as it will engage the procedural memory system. At the time of retrieving the information, the exact experience will be confounded with other average experiences. First, this result suggests more generally that we do not remember all events similarly and we use more memory resources to remember non ordinary events (such as emotional memories or flashbulb memories). This description is in line with the neuroscience evidence suggesting that the declarative system is invoked to remember striking, episodic events whereas the procedural system is used to remember general aspects and recurrent patterns of tasks (see section 1). Second, and more importantly, our paper argues that these are not exogenous properties of the memory systems. Instead, they are the result of an optimal memory management policy. So, for example, we optimally remember the details of a clever proof, the ingredients of an awesome desert, and the moment where the music conductor made a mistake. And at the same time, we optimally keep only a vague idea of what was in a moderately interesting paper, in an uninspiring meal and in an average concert. Overall, our model suggests that memory accuracy is due to the endogenous selection of different systems to retrieve average vs. extreme experiences.

**Implication 1** *Efficient memory management requires striking events to be retrieved with the declarative system and non striking events to be retrieved with the procedural system. Striking events are then vividly remembered while non striking events remain blurry.*

*2. Memory impairment and biased behavior.*

Our theory builds on the premise that the declarative and procedural systems are substitutable, an idea that has received experimental support in neuroscience (see section 1). Sometimes, however, substitution is impossible. Suppose for instance that one memory system is not functioning properly (e.g., due to a lesion or task overload), in which case memories are routed inefficiently to the only available system. When the declarative system is used, DM's behavior tends to hinge closer to the signal compared to when the procedural system is used. In both cases, experiences $x < \mu$ trigger actions that are, on average, below $\mu$ and experiences $x > \mu$ trigger actions that are, on average, above $\mu$. However, when comparing the behavior of subjects with different impairments, we notice a systematic bias in decision-making. Formally, the decision taken on average is:

$$E[\hat{a}(s_i)] = \frac{p}{p+h_i}\,\mu + \frac{h_i}{p+h_i}\,x$$

and the bias of the expected decision with respect to the original experience is:

$$E[\hat{a}(s_i)] - x = \frac{p}{p+h_i}\,(\mu - x)$$

As the precision of the memory increases, the average decision comes closer to the original experience. Formally, $|E[\hat{a}(s_i)] - x|$ is decreasing in $h_i$. Therefore, subjects exhibiting an impaired procedural memory and resorting exclusively to the declarative memory (the high precision system) are more likely to behave in accordance with the original experience. By contrast, subjects with an impaired declarative memory will tend to depart more from the original experience. This implication indicates that subjects with deficits of the hippocampus and related structures should perform poorly at remembering "whether they liked the product". This prediction is in line with experimental evidence (Eichenbaum et al., 1990). By contrast, subjects with a deficit of the striatum should recall all types of experiences equally well. Overall, in the limit case where a system has become infinitely costly to use (a simple way of modeling an impairment), memories will be encoded and retrieved inefficiently and behavior will be systematically biased.

**Implication 2** *DM with impairment of the procedural system tends to form very precise memories and act upon a precise recollection. Behavior over time responds closely to past experiences. By contrast, DM with impairment of the declarative system tends to form very vague memories of events and act upon the prior. Behavior over time does not respond to experiences.*

*3. Environmental factors and memory-based behavior.*

Naturally, even in the absence of impairments, we do not always recall similar events equally well. The day-to-day demands imposed on systems make them relatively more or less costly to use. Changes in our environment may act as triggers to increase the cost of storing memories or to make us remember less accurately. For instance, it has been shown that psychological stress affects hippocampal functions and declarative memory performance (Lupien et al. (1997), Henckens et al. (2009)). Not surprisingly, $dx^*/dc_D > 0$ and $dx^*/dc_P < 0$ and, for precision values large enough, $dx^*/dh_D < 0$ and $dh^*/dh_P > 0$ (a sufficient condition is $h_P > p$). A system is relatively less likely to be employed the higher its associated cost and the smaller its accuracy. Therefore, under stress, when normal hippocampal functions are disrupted, it is optimal to form new memories through the procedural system even when events are striking. This obviously results in imperfect memories.

**Implication 3** *Day-to-day memory formation is affected by the relative demands imposed on memory systems as we substitute them to use the relatively more efficient one.*

*4. Memory and incentives.*

Some events are intrinsically more important to us than others. Recall that $\beta$ reflects the sensitivity to losses, with a larger $\beta$ implying a steeper loss function. When $\beta$ increases, it becomes more valuable to recall information accurately. Therefore, DM should use the declarative system more often ($\partial x^*/\partial\beta < 0$). Thus, according to the model, memories are shaped by incentives. Moreover, from Implication 3, we know that as the cost of one memory system increases, the likelihood of a substitution with the other system also increases. Interestingly, this substitution effect is as small as the cost of suboptimal choices increases. That is, subjects who lose more from imperfect recall are also less sensitive to variations in the cost of a system: they tend to switch less often when the system becomes more effortful.

**Implication 4** *When suboptimal actions are more costly, DM resorts less often to the procedural system. Furthermore, he is less likely to substitute memory systems if their relative cost change.*

*5. Memorizing unexpected events and learning new material.*

Sometimes, we face events for which the best prior we can formulate is flat. This occurs in the case of new events or unpredictable and novel situations. In those cases, the variance is large ($p$ is small) and DM relies almost exclusively on the signal, that is, the decision is close to $s_i$. Recall that the signal is closer to the experience under the declarative system but it is also more costly. Thus, when $p$ is small, it is optimal to choose the system that offers the best trade-off between cost and precision, independently of the realization of $x$. Some DM will resort to the procedural system while another will invoke the declarative system. However, the recording pattern for a given DM is predicted to be the same across events (product A vs. product B) or as a function of the realized state (I liked it a lot vs. I liked it moderately). Interestingly, this result suggests the existence of different learning patterns. We should expect a DM who resorts to the procedural system to learn about unpredictable situations very slowly (as a result of not remembering well). By contrast, a DM who resorts to the declarative system should learn very quickly if exposed repeatedly to novel situations. In the same lines, consider a child who learns how to read and has no prior knowledge of the alphabet. When exposed to a letter for the first time, he may forget it, making mistakes repeatedly. The child is (possibly optimally) using the low-cost procedural memory and learns more slowly than a child who uses the (effortful) declarative memory. Even though our model is not designed to make predictions about learning (which would require a dynamic framework), it points to the existence of a causal relationship between the ability to learn/memorize and the intrinsic features of our two memory systems.

**Implication 5** *When our beliefs are flat, either the procedural system is always used or the declarative system is always used.*

# 5 Conclusion and future directions

In this article we have incorporated the evidence from neuroscience regarding the existence of multiple memory systems and built a theory able to unveil causal relationships between memory systems and the characteristics of the information retrieved. The theory argues that such causal relationships emerge as the result of an optimal memory management policy. The type of information memorized by each system is the solution to a cost-benefit trade-off between effort required and precision obtained of memories. The theory also provides a coherent and unified framework to understand behavioral biases attributable to imperfect memory.

Despite the significant evidence in favor of multiple memory systems reviewed in the introduction, there is still a certain resistance to the concept (see e.g. Nosofsky and Zaki (1998) and Palmeri and Flanery (1999)). Sherry and Schacter (1987) propose an interesting evolutionary theory, where the development of multiple memory systems is driven by the incompatible needs to learn common features across events vs. learning specific features of each event. The argument is attractive but lacks a formal mathematical framework. Developing rigorous models that capture the evolutionary value of internal conflicts (as, for example, Samuelson and Swinkels (2006) or Bisin and Iantchev (2010) do in other contexts) could help clarify the relative advantages of one vs. multiple memory systems.

Traditional economic theories of learning could benefit from modeling biological mechanisms to rationalize biases in beliefs and behavior (Brocas and Carrillo, 2012). For instance, the existence of multiple memory systems is likely impacting belief formation as a function of the environment in which learning takes place. In rapidly changing environments, where information is highly valuable but becomes obsolete quickly, individuals should optimally employ the costly but accurate declarative memory system. By contrast, in stable settings it might be preferable to save on cost and invoke the imprecise procedural system, hence reducing the speed of learning. Furthermore, given rare and striking events are more often remembered, predictions regarding future similar events may be more accurate compared to predictions regarding more common events. This could explain why we act as if we overweight low probability (rare) events compared to others. Also, events that are rare at a point in time but become common after sufficient exposure should be memorized differentially over time, leading to biases in the decisions we make and that are usually thought to be changes in utility. More generally, understanding better how we encode, memorize and retrieve the information we gather around us should shed light on what we believe and how we behave given these memories.

Finally, the study also provides an example of the mirror cross-fertilization possibilities, namely how the methodology in microeconomic theory can help understand biological phenomena and offer new testable predictions. Yet, the theory presented here abstracts from important considerations. Of special interest is the evidence supporting the idea that different memory systems impose ‘externalities’ on each other. For instance, Gold (2004) shows that memory systems may compete or cooperate with each other in certain situations. In future work, it would be interesting to theoretical model and empirically test such interactions.

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# Appendix

**Proof of Implication 3**. At equilibrium $V_P(\mu - x^*) = V_D(\mu - x^*)$. Differentiating this expression with respect to $c_P$ we have,

$$-\Big(\frac{\partial V_P}{\partial x}(\mu - x^*) - \frac{\partial V_D}{\partial x}(\mu - x^*)\Big)\frac{\partial x^*}{\partial c_P} = -\frac{\partial V_P}{\partial c_P}(\mu - x^*).$$

We also have $\frac{\partial V_P}{\partial x}(\mu - x^*) - \frac{\partial V_D}{\partial x}(\mu - x^*) \propto h_D - h_P > 0$ and $\frac{\partial V_P}{\partial c_P}(\mu - x^*) = -1$, therefore, $\frac{\partial x^*}{\partial c_P} < 0$. As $c_P$ increases, $\mu - x^*$ increases and $\mu + x^*$ decreases: the region in which DM uses the procedural memory shrinks. Similarly,

$$-\Big(\frac{\partial V_P}{\partial x}(\mu - x^*) - \frac{\partial V_D}{\partial x}(\mu - x^*)\Big)\frac{\partial x^*}{\partial c_D} = \frac{\partial V_D}{\partial c_D}(\mu - x^*)$$

and given $\frac{\partial V_D}{\partial c_D}(\mu - x^*) = 1$, we have $\frac{\partial x^*}{\partial c_D} > 0$. With respect to precisions, we have

$$-\Big(\frac{\partial V_P}{\partial x}(\mu - x^*) - \frac{\partial V_D}{\partial x}(\mu - x^*)\Big)\frac{\partial x^*}{\partial h_P} = -\frac{\partial V_P}{\partial h_P}(\mu - x^*),$$

$$-\Big(\frac{\partial V_P}{\partial x}(\mu - x^*) - \frac{\partial V_D}{\partial x}(\mu - x^*)\Big)\frac{\partial x^*}{\partial h_D} = \frac{\partial V_D}{\partial h_D}(\mu - x^*)$$

Note that $\frac{\partial V_i}{\partial h_i}(\mu - x^*) \propto h_i - p + p^2 x^* > 0$ if $h_i > p$ in which case $\frac{\partial x^*}{\partial h_P} > 0$ and $\frac{\partial x^*}{\partial h_D} < 0$.

**Proof of Implication 4**. Differentiating the equilibrium condition $V_P(\mu - x^*) = V_D(\mu - x^*)$ with respect to $\beta$ we have,

$$-\Big(\frac{\partial V_P}{\partial x}(\mu - x^*) - \frac{\partial V_D}{\partial x}(\mu - x^*)\Big)\frac{\partial x^*}{\partial \beta} = -\Big(\frac{\partial V_P}{\partial \beta}(\mu - x^*) - \frac{\partial V_D}{\partial \beta}(\mu - x^*)\Big)$$

At equilibrium, we have $\frac{\partial V_P}{\partial \beta}(\mu - x^*) - \frac{\partial V_D}{\partial \beta}(\mu - x^*) = (c_P - c_D)/\beta < 0$ and therefore $\frac{\partial x^*}{\partial \beta} < 0$. Differentiating a second time with respect to $c_D$:

$$\Big(\frac{\partial^2 V_P}{\partial x^2}(\mu - x^*) - \frac{\partial^2 V_D}{\partial x^2}(\mu - x^*)\Big)\frac{\partial x^*}{\partial \beta}\frac{\partial x^*}{\partial c_D} - \frac{1}{\beta} = \Big(\frac{\partial V_P}{\partial x}(\mu - x^*) - \frac{\partial V_D}{\partial x}(\mu - x^*)\Big)\frac{\partial^2 x^*}{\partial \beta \partial c_D}$$

yielding $\frac{\partial^2 x^*}{\partial \beta \partial c_D} < 0$. Differentiating now a second time with respect to $c_P$:

$$\Big(\frac{\partial^2 V_P}{\partial x^2}(\mu - x^*) - \frac{\partial^2 V_D}{\partial x^2}(\mu - x^*)\Big)\frac{\partial x^*}{\partial \beta}\frac{\partial x^*}{\partial c_P} + \frac{1}{\beta} = \Big(\frac{\partial V_P}{\partial x}(\mu - x^*) - \frac{\partial V_D}{\partial x}(\mu - x^*)\Big)\frac{\partial^2 x^*}{\partial \beta \partial c_P}$$

yielding $\frac{\partial^2 x^*}{\partial \beta \partial c_P} > 0$.

**Proof of Implication 5**. When $p \to 0$, we have $V_i(x) = -\frac{\beta}{h_i} - c_i$ and $V_D(x) - V_P(x) = -\frac{\beta}{h_D h_P}(h_P - h_D) + c_P - c_D$, which is independent of $x$.